The total work is

$$A = \int_{r_1}^{r_2} dA = \frac{qq_0}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{r_1} - \frac{qq_0}{r_2}\right). \qquad (2.9)$$

From this equation we can see that a work done doesn't relate to the shape and length of the path *L*. A work is defined only by the position of the points *1* and *2*. Consequently, the electric field of a unit charge is potential and electrostatic force is conservative. The work, done by this electrostatic forces around any closed contour is always zero: $(\oint dA = 0)$. That is

$$\oint q_0 E dl = 0 . \qquad (2.10)$$

And for unit charge $q_0 = 1$:

$$\oint E dl = 0. \qquad (2.11)$$

This integral is called ***the intensity vector*** $\vec{E}$ ***circulation***. Consequently, the circulation of the vector intensity of electrostatic field around any closed contour is always equal to zero. The field, that has this property, is called potential.

Potential is the energetic characteristic of the electric field. It is denoted as φ . The potential at a given point is the potential energy of a unit positive charge, placed at this point:

$$\varphi = \frac{W}{q_0}. \qquad (2.12)$$

It is a scalar function of coordinates:

$$\varphi = \varphi(x, y, z) = \varphi(r). \qquad (2.13)$$

The unit of potential is called Volt. $[\varphi] = \frac{\mathrm{J}}{\mathrm{C}}.$

The work which electric field performs, removing unit positive charge q_0 from point *1* to point *2* is:

$$A = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = W_1 - W_2. \qquad (2.14)$$

The work is equal to the difference of potential energies of charge q_0 at the point 1 and 2:

$$\varphi = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r};$$

$$A = q_0(\varphi_1 - \varphi_2). \qquad (2.15)$$

If unite positive charge removes from given point to infinity, which $\varphi = 0$, the work

$$A_\infty = q_0 \varphi;$$

and

$$\varphi = \frac{A_\infty}{q_0}. \quad (2.16)$$

This is the following definition of potential: the work, electric field performs, removing unit positive charge from given point to infinity is called the potential of an electric field at a given point. At the given point the potential of a set of point charges is the algebraic sum of the potentials, created at the given point by each charge separately:

$$\varphi = \sum_i \varphi_i. \quad (2.17)$$

Potential plays the same role for the charge that the pressure does for the fluids. If there is a pressure difference between two ends of a pipe filled with the fluid, the fluid will flow from the high pressure end towards the lower pressure end. Charges respond to differences in potential in a similar way.

Work and potential energy in an electric field

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

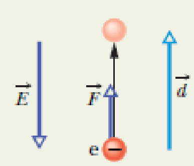

Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \quad (24\text{-}3)$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos\theta, \quad (24\text{-}4)$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180° = 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \quad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.

Net potential of several charged particles

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \quad q_3 = +31\text{ nC},$$
$$q_2 = -24\text{ nC}, \quad q_4 = +17\text{ nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.

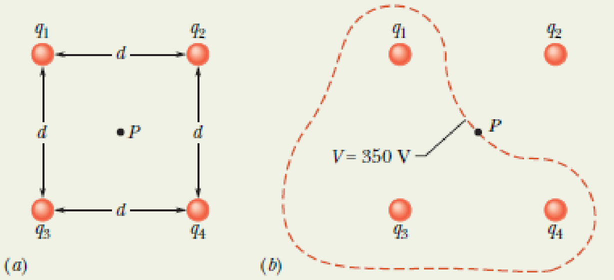

Fig. 24-8 (a) Four point charges are held fixed at the corners of a square. (b) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point P. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\text{ C}$$
$$= 36 \times 10^{-9}\text{ C}.$$

Thus,
$$V = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9}\text{ C})}{0.919\text{ m}}$$
$$\approx 350\text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8a, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point P. The curve in Fig. 24-8b shows the intersection of the plane of the figure with the equipotential surface that contains point P. Any point along that curve has the same potential as point P.

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

Calculations: Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say q_1, in place and the others at infinity. Then we bring another one, say q_2, in from infinity and put it in place. From Eq. 24-43 with d substituted for r, the potential energy U_{12} associated with the pair of point charges q_1 and q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge q_3 in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring q_3 near q_1 and the work we must do to bring it near q_2. From Eq. 24-43, with d substituted for r, that sum is

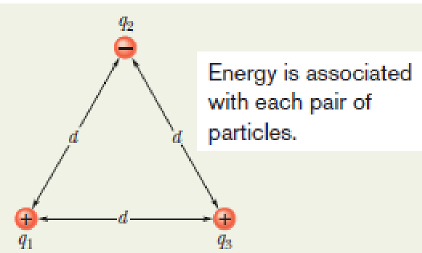

Fig. 24-16 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$U = U_{12} + U_{13} + U_{23}$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$
$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$
$$= -\frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\text{ C})^2}{0.12\text{ m}}$$
$$= -1.7 \times 10^{-2}\text{ J} = -17\text{ mJ}. \qquad \text{(Answer)}$$

Electric Capacitance

A capacitor is a device for storing charge. It is usually made up of two plates separated by a thin insulating material known as the dielectric. One plate of the capacitor is positively charged, while the other has negative charge. The electric capacitance is a property of any conductor. This property provides an ability to accumulate the electric charges. By definition the ratio

$$C = \frac{q}{\varphi} \qquad (3.22)$$

is called the electric capacitance of an isolated conductor. Isolated conductor is located sufficiently far from other bodies. The charge of a body under consideration is q, its potential is φ. As we know for a point charge:

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}. \qquad (3.23)$$

Thus, $$\varphi \sim q; \qquad (3.24)$$

and $$\varphi = \frac{1}{C} q \,, \qquad (3.25)$$

where *C* is the coefficient of proportionality. Value of C is an electric capacitance. The SI unit of capacitance is:

$$[F] = \frac{C}{V}.$$

The capacitance of a charged sphere can be calculated as:

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R},$$

where *R* is the radius of the sphere. By definition:

$$C = \frac{q}{\varphi} \qquad (3.26)$$

and from the last equation we obtain

$$C = 4\pi\varepsilon_0 R. \qquad (3.27)$$